ASSESSING VIBRATING VISCOMETERS

IN SLURRIES.

by

Y.A.RACHMAN

Submitted to the University of Cape Town in
fulfilment of the requirements for the
degree of Master of Science in
Engineering, September 1977.

ACKNOWLEDGEMENTS

The author wishes to express appreciation and gratitude to his supervisor, Professor J.L. Besseling, for the interest he has displayed towards this project, and the advice and guidance he has given throughout its duration. Also to Dr. D. Naudé for his valued help and encouragement and to the staff of the Central Acoustics Laboratory for their friendship and willingness to be of assistance.

All of the above have contributed towards making this period in the author's life a most pleasant and memorable one.

Thanks is also expressed to the National Institute for Metallurgy who sponsored this research in the form of a bursary.

SUMMARY.

A number of vibrating transducers for viscosity measurement described in the literature have been evaluated, with a view to developing an instrument for measuring gold-mine slurries.

Appropriate electronic measuring methods are discussed, and related theory using a simple mathematical model is derived, and verified experimentally.

A low frequency vibrating transducer is described which is capable of measuring apparent viscosities of slurries. Measurements made on Newtonian liquids at four different frequencies using the same transducer agree to $\pm$ 2%, and correlate closely with measurements made using a Redwood orifice viscometer.

In order to determine the accuracy of the transducer for slurries, comparative measurements at four frequencies are used to obviate the need for a reference instrument.

For a dilute (Newtonian) slurry, these measurements agree to $\pm$ 10%, but similar measurements on more concentrated (non-Newtonian) slurries show marked divergence of the curves at high frequencies. It is suspected that this behaviour is a property of the slurries, related in some way to their observed non-Newtonian behaviour. These results are of a qualitative nature, but point to the possible usefulness of the instrument for slurry measurement.

INTRODUCTION.

The use of vibrational methods for the measurement of viscosity has long been known and commercial vibrating viscometers have been available for at least twenty years. They seem, however, to have found their main application in the food, petroleum and polymer industries.

A review of several papers, desribing vibrating viscometers, revealed that although a number of the workers had taken measurements on suspensions, only one (7) did not use a relative scale and even this paper did not claim that the readings obtained were accurate.

In the metallurgical industry there are many applications for such measurements and especially important are measurements on slurries and heavy media. Owing to their mode of operation vibrating probes cause a negligible disturbance to the process being investigated; a property particularly useful in establishing density gradients in settlers and heavy-medium or magnetic-liquid separators.

The aim of this project was:

(1) To investigate various electrical measuring techniques which can be used to find the loss factor of the immersed viscosity probe, using Newtonian fluids for calibration purposes.

(2) To design suitable probes with which to establish whether a vibrating probe can be used for measuring the apparent viscosity of slurries.

(3) To verify some theoretical predictions with reference to relevant literature.

(4) To make recommendations as to the design of a suitable instrument.

A number of probes, for measuring viscosity, were constructed and tested. Each such probe was used for increasingly exacting measurements until a stage was reached when a more sophisticated probe was required. At the same time, measuring techniques were improved, and suitable theories deduced for these.

Satisfactory measurements on slurries were possible in the closing stages of the project when the problems encountered with getting repeatable results for slurries had been overcome.

The probe developed for the slurry viscosity measurements is also able to measure density of some liquids.

A method for evaluating the behaviour of a vibrating probe in a slurry is described, which obviates the need for calibrating by a measurement on the slurry using a standard viscometer.

TABLE OF CONTENTS.

CHAPTER 1. VISCOSITY MEASUREMENT: A REVIEW.

1.0 Viscosity

"Viscosity may be thought of as the internal friction of a fluid. Because of viscosity, a force must be exerted to cause one layer of a fluid to slide past another, or to cause one surface to slide past another if there is a layer of fluid between the surfaces"(11).

The force per unit area (ie. stress) exerted is proportional to the rate of shear, which is the change in velocity per unit distance from a surface.

Mathematically,

$$\sigma_S = -\eta \frac{\partial u_x}{\partial y}$$

Where σ_S is the shear stress. (force per unit area).
u_x is the velocity of the fluid (along the X-axis).
y is the axis normal to the surface.
η is the coefficient of viscosity.
η has units (dyne-sec/cm) in the c.g.s. system.
1 dyne-sec/cm = 1 poise.

A Newtonian liquid is one for which η is a constant, independent of shear rate. Water is one such a liquid.

Any liquid for which η is a variable dependent on shear rate is classified as non-Newtonian. The liquid is further classified according to the nature of this variation, as shown in Fig. 1.1.

1.1 The Ideal Viscometer

Here an ideal viscometer specification is proposed, to be used as a standard for evaluating actual viscometers.

The ideal viscometer:

1. Is non-intrusive. ie. it does not disturb the test liquid in any way.

2. Is not affected adversely by the test liquid. For example it is corrosion and abrasion resistant, and mechanically robust.

3. Gives correct readings independent of pressure, temperature or other extraneous factors. (including noise).

4. Is fully immersible to allow viscosity measurements in the body of the fluid.

5. Requires a small sample of liquid for the measurement, both for convenience and to facilitate the making of point measurements.

6. Is able to measure non-Newtonian liquids, and to give correct readings irrespective of the liquid composition.

7. Has a variable shear rate to enable the viscosity of non-Newtonian liquids to be evaluated as a function of shear rate.

1.2 Review of Existing Viscometers and Reason for Choice of Vibrating Viscometer.

Most viscometers in use today use either steady flow methods in order to measure viscosity, (e.g. orifice viscometers) or measure the viscous drag on a body moving through the liquid (e.g rotating and falling ball viscometers). All the methods are intrusive, many are slow, and the viscometers are often delicate instruments. (Ref. 12).

Vibrating viscometers, on the other hand, can be made very robust, and the amplitude of vibration made sufficiently small to ensure that the device is non-intrusive. Also, a continuous readout is possible.

1.3 Literature Review of Vibrating Viscometers.

Mason (1) first demonstrated how the viscosity of a fluid can be measured sonically. A tubular crystal oscillating in a torsional mode was used to shear the test fluid in a direction tangential to the crystal surface, thus setting up a shear wave in the fluid. The resultant loading of the crystal comprises:

(a) A resistive component due to the viscous drag.

(b) A reactive component due to the mass of the fluid which moves with the crystal, and the shear elasticity of the fluid if its behaviour is non-Newtonian at the frequency of oscillation.

This loading can be measured by the resultant change in electrical resistance and resonant frequency of the crystal.

One of the crystals used did not oscillate in pure torsion, resulting in longitudinal waves being generated as well. Mason described a method for reflecting this longitudinal radiation back to the crystal, so that any resistive loading of the crystal could be attributed to radiation of shear waves only.

Measurements made for light Newtonian oils with viscosities ranging from 0,01 to 10 poise were accurate to within five percent. However, measurements made for very viscous liquids such as polymerised castor oil and vistanex showed that these liquids possess shear elasticity at high frequencies, although at low frequencies their behaviour is Newtonian. This verifies Maxwell's hypothesis that viscosity is the product of a shear elasticity by a relaxation time.

McSkimin (2) used a torsionally oscillating quartz crystal connected at one end to long glass or metal rod. A short periodically repeated train of torsional waves was transmitted by the crystal along the rod, the signal reflected from the free end being received by the same crystal. Immersing a portion of the rod in the test liquid resulted in a change in attenuation and phase of the received signal, from which the viscosity and shear rigidity could be deduced.

A detailed description was given of the circuitry used. A number of very viscous liquids were tested using this apparatus, the results showing the presence of shear elasticity in these liquids at high frequencies, again supporting Maxwell's hypothesis. Two frequency ranges were used:- 25-55 KHZ, and 60-140 KHZ.

The advantages claimed for the system are:

(a) It is possible to separate in time the desired torsional signal from any spurious longitudinal signals generated by the crystal, as the latter have a higher propagation velocity, and will therefore arrive at the receiver first.

(b) The system does not operate at resonance, hence the length of the rod can be chosen for convenience, and the system will operate over a wide frequency band.

A major drawback for slurry measurement is that the design is not readily adapted to low frequency operation.

Roth and Rich (3) developed the "Ultraviscoson", a commercial viscometer which used as the transducer a thin flat magnetostrictive blade vibrating longitudinally, so that the test fluid was sheared in a direction parallel to the plane of the blade.

An excitation coil was wound around one half of the blade, which was encapsulated in a sheath connected to the centre (nodal) point of the blade. The half of the blade which protruded from the sheath was immersed in the test liquid.

The viscous loading on the transducer was measured by first exciting the transducer at a suitable resonant frequency, then removing the excitation and observing the resultant damped oscillations of the transducer. For this purpose, the excitation coil was used both as a transmitter and as a receiver. This type of measuring technique is simple to automate, but is only accurate for light damping.

In deriving the theory of their transducer, Roth and Rich maintained that the ends and edges of the blade have negligible area compared to the flat surfaces, and "so will affect the results to a negligible degree". This may be true for the edges of the blade, but the end immersed in the liquid will radiate longitudinal waves, the loading of which, according to Taraba (4), can be neglected only if the ratio of the flat surface area to the frontal area is of order 10^4 or more. Also, as the blade vibrates longitudinally, its thickness and width vary due to the Poisson coupling, resulting in longitudinal radiation normal to the flat surfaces and the edges.

They performed a detailed analysis of the transducer. Measurements using the apparatus were made on thin viscous films, human blood, and colloidal Bentonite suspensions, as well as a number of polymers.

They point out that with suspensions, the particles tend to migrate to boundaries where they reduce surface tension. If they migrate to the transducer, an erroneous reading results. Fortunately, this tendency is easily countered by agitating the liquid.

An accuracy of $\pm$ 2% of full scale was claimed for the instrument.

Papers by several other workers (4, 5, 6,) mainly describe improvements to above mentioned devices, develop the theory in greater detail, or describe specific applications.

Taraba (4) describes a modified version of the transducer developed by Roth and Rich (3). He recognized that the frontal loading of a longitudinally vibrating blade could not be reduced sufficiently to be neglected. His solution to the problem was to lengthen the transducer by a half-wavelength, enclosing the last quarter wavelength in a box connected to the nodal point. In this way, the end of the transducer radiated into the air in the box, a small and constant factor.

However, no mention was made of the variation in thickness of the transducer, and the resultant longitudinal radiation. An accuracy of ± 4,5% was claimed for the instrument.

Papers by Bradfield (5) and Ruttle and Stephenson (6) both describe torsionally vibrating ultrasonic viscometers similar in operation to Mason's (1), but with modifications making them suitable for use at high pressures. They are therefore of little interest to the present project.

Woodward (7) described a low frequency transducer that was simple to construct. It comprised a flat disc vibrating in its own plane on the end of a reed. The reed was excited by a magnetic drive assembly, and the resultant vibration was picked up via two crystals soldered on opposite faces of the steel reed. The transducer formed part of a self-oscillating circuit, so that it always operated at a resonant frequency. The drive was arranged to be constant, so that the output voltage from the pickup, which was proportional to the amplitude of vibration, decreased with increased damping of the transducer. In operation, the disc was immersed in the test liquid, shearing it in the plane of the disc.

This device generates no longitudinal waves, because the radiation from the front and rear edges of the disc is out of phase, and so cancels.

It is necessary for the disc to be immersed to a critical depth. This:

(a) Reduces the reproducibility of readings.

(b) Limits the device to surface measurements.

Kremlevskii and Stepichev (8) describe a low-frequency transducer capable of measuring both viscosity and density. Their transducer comprises a uniform rod of circular cross-section vibrating in a flexural mode. When partially immersed in the test liquid, the rod displaces liquid as it moves. This movement results in viscous loading of the rod similar to that experienced by the transducers already discussed, plus an additional mass loading of the rod due to the displacement of the liquid. This mass loading results in a greater change in resonant frequency of the rod than would occur for pure viscous loading, enabling the density of the liquid to be calculated.

No longitudinal waves are radiated by the rod, because its diameter is much less than a wavelength at the operating frequency.

A detailed derivation of the theory of the transducer is performed, and verified experimentally, the measurement error being less than 2%. The measuring technique is similar to that used by Roth and Rich (3).

This transducer is of great relevance to the present project, because:

(a) It can measure both viscosity and density.

(b) Its frequency of operation is low.

(c) It can be made fully immersible.

A paper by Kogan (9) discusses a method of lowering the frequency of operation of tubular longitudinal or torsional transducers by the addition of lumped masses to the ends. He also analyses the error introduced by the frontal loading of the longitudinal transducer, showing that it is significant at low viscosities. He neglects any longitudinal radiation due to the variation in thickness of the transducer, maintaining that its effect is small compared with the frontal loading. This may not be the case for a longitudinal transducer such as that of Roth and Rich (3) (where the frontal area is small), or Taraba (4) (where the frontal loading has been removed).

Krutin and Smirnitskii (10) recognize that all the transducers used for viscosity measurement are essentially the same. They perform a general analysis in order to optimize the choice of transducer parameters and measuring technique for specific applications, with a view to minimizing measurement errors.

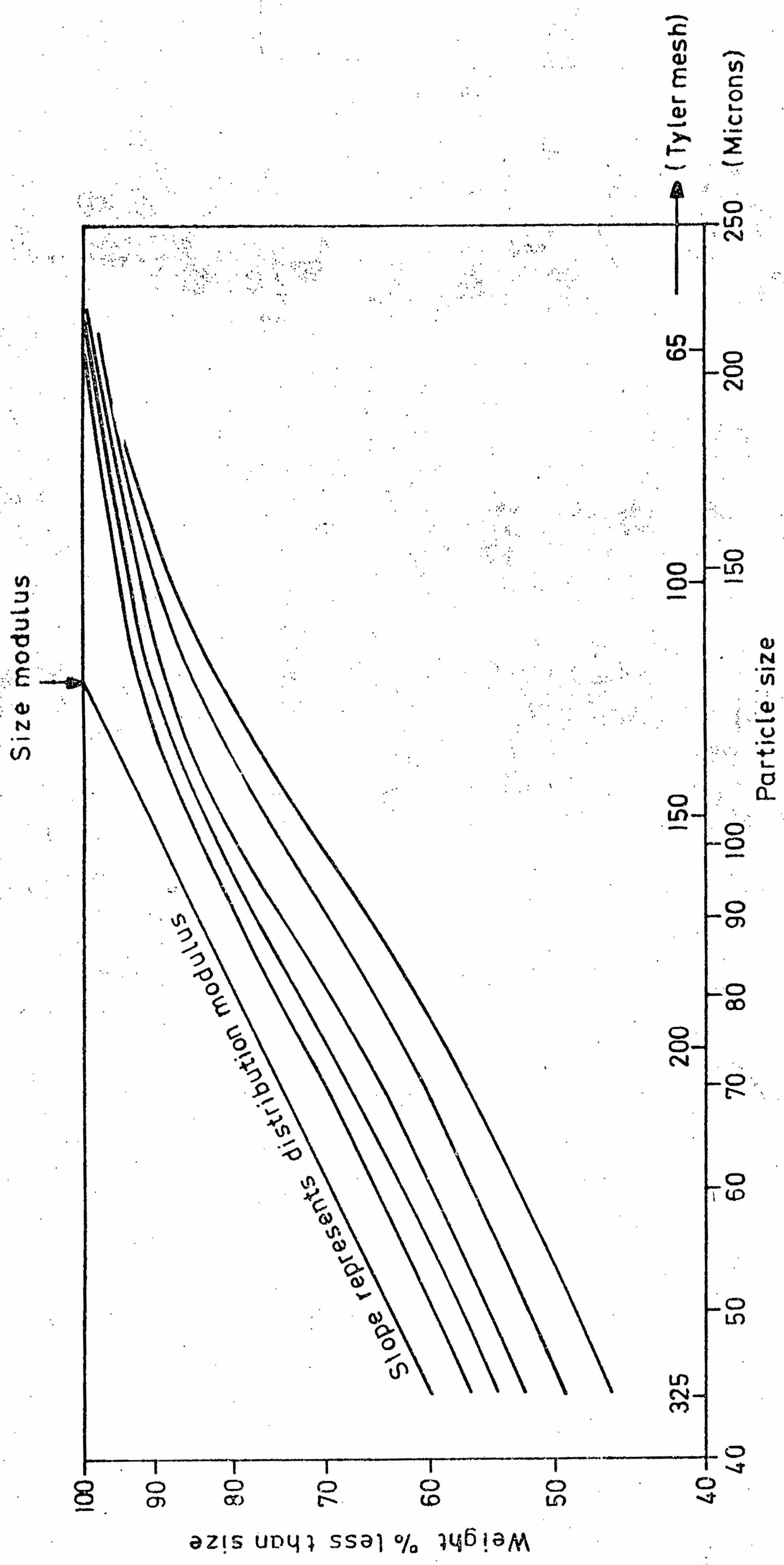
1.4 A Typical Gold-Mine Slurry.

A typical gold-mine slurry comprises milled quartzite suspended in water. The pH can range from approximately 2 to 11, and the temperature from $8^{o}C$ to $80^{o}C$. A cumulative particle size distribution curve is shown in fig. (1.2). (From ref. 13).

Results by Levy (14) who investigated the rheological properties of a number of South African gold-mine slurries, using a rotating viscometer, showed that these slurries are non-Newtonian.

It is apparent that any instrument used to measure the apparent viscosity of these slurries must satisfy the following requirements:

1. Abrasion, temperature and corrosion resistant.
2. Measurement unaffected by the particulate (ie non-homogeneous) nature of the slurry.
3. Variable shear rate, so that the non-Newtonian behaviour can be investigated.



Size modulus
Slope represents distribution modulus
Weight % less than size
100
90
80
70
60
50
40
Particle size
(Tyler mesh)
65
100
150
200
325
(Microns)
250
200
150
100
90
80
70
60
50
40

Figure 1.2

1.5 Aim of Project.

The aim of this project was as follows:

1. Develop a transducer capable of measuring viscosities of homogeneous Newtonian liquids.

2. Develop a suitable measuring technique to give repeatable results consistent with theoretical predictions.

3. Calibrate the transducer on Newtonian liquids, using a standard viscometer as a reference.

4. Apply the measuring techniques to slurries, with the emphasis on repeatability of readings.

5. Once repeatable readings were obtained, to establish whether these readings were correct.

6. Take note of whether the limited depth of penetration of the shear wave affected readings on slurries.

CHAPTER 2. THEORY

2.1 Specific Shear Impedance of Liquid

When an infinite plane (ie no end effects) in contact with a Newtonian liquid of viscosity η and density ρ vibrates sinusoidally at a frequency f with velocity u it experiences a shear force per unit area given by

$$\sigma_S = (\pi f \rho \eta)^{1/2} (1+j)\, u$$

This is usually written in the form

$$Z_S = (\pi f \rho \eta)^{1/2} (1+j) \qquad 2.1$$

where $Z_S = \sigma_S / u$ is the specific shear impedance of the liquid. This expression is derived in appendix B.

Note that Z_S has equal resistive and reactive components, the latter due to the mass of liquid which moves with the plane.

Although the above expression was derived for an infinite plane, it is generally accepted as valid provided the plane is large compared with a shear wavelength. (This condition is easily satisfied in practice)

2.2 Penetration of the Shear Wave into the Liquid.

It can be shown (see appendix B) that for a Newtonian liquid, the absorption coefficient for shear waves is given by:

$$\alpha_S = \left(\frac{\pi f \rho}{\eta}\right)^{1/2} \qquad 2.2$$

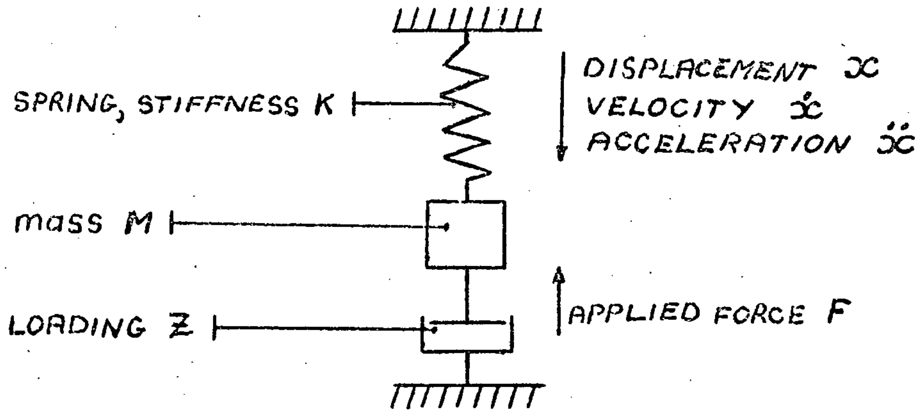
For water, the value of the absorption coefficient at a frequency of 10 KHz is $1{,}8 \times 10^3$ nepers/cm. This is extremely high, resulting in a very rapid attenuation of the waves, so that they penetrate only a very short distance into the medium.

It can be shown (app. B) that the waves are attenuated to approximately 1% of their initial amplitude within one wavelength. (30 μm at 10KHz in water)

2.3 Transducer Modelled as a Mass-Spring System.

In order to gain an insight into the workings of the transducers used, a simple mathematical model was used to describe them. The transducers used, although all distributed systems, can be modelled at frequencies near a resonance as simple mass-spring systems, provided that their Q-factors remain sufficiently high.

Consider a simple mass-spring system, with viscous damping, and undergoing forced oscillation:



For equilibrium,

$$m\ddot{x} + Z\dot{x} + Kx = F \qquad 2.3$$

From equation (2.1), assuming sinusoidal motion,

$$Z = R(\omega) + jX(\omega) \qquad 2.4$$

for a Newtonian fluid, where

$$R(\omega) = A(\rho\eta\omega)^{1/2} + R_0$$

and A is the contact area (for a flat surface).
ρ is the fluid density.
ω is the angular frequency.
R_0 is the transducer loss in a vacuum.

$$X(\omega) = A(\rho\eta\omega)^{1/2}$$

If F is sinusoidal, ie $F = F_0 e^{j\omega t}$
then, since the system is linear, x is sinusoidal.

i.e. $x = x_0 e^{j\omega t}$

$$\dot{x} = j\omega x$$

$$\ddot{x} = -\omega^2 x$$

substituting in equation 2.3,

$$K - \omega^2 m - \omega A(\rho\eta\omega)^{1/2} + j\omega R_0 + j\omega A(\rho\eta\omega)^{1/2} = \frac{F}{x} \qquad 2.5$$

2.4 Finding $\rho\eta$ by Measuring Relative Magnification.

At resonance, by definition the spring and mass terms cancel, so

$$j\omega_0\left[A(\rho\eta\omega_0)^{1/2} + R_0\right] = \frac{F_0}{x_0} \qquad 2.6$$

where ω_0 is the immersed resonant frequency.
x_0 is the amplitude at resonance.
F_0 is the force at resonance.

now, a vacuum, ie no liquid, $\rho = 0$

$$\frac{F_{00}}{x_{00}} = j\omega_{00}R_0 \qquad 2.7$$

where ω_{00} is the free resonant frequency.
x_{00} is the amplitude for this case.
F_{00} is the force for this case.

Dividing equation 2.6 by 2.7,

$$\frac{F_0 x_{00}}{F_{00} x_0} = \frac{\omega_0}{\omega_{00}}\frac{A}{R_0}(\rho\eta\omega_0)^{1/2} + 1$$

i.e.

$$\rho\eta = \frac{R_0^2}{A^2\omega_0}\left[\frac{\omega_{00} F_0 x_{00}}{\omega_0 F_{00} x_0} - 1\right]^2 \qquad 2.8$$

$$\rho\eta = \frac{C}{f_0}\left[\frac{f_{00} F_0 x_{00}}{f_0 F_{00} x_0} - 1\right]^2 \qquad 2.8(a)$$

where $C = \frac{2\pi R_0^2}{A^2}$ is a constant.

2.5 Finding $\rho\eta$ by measuring bandwidth.

From equation 2.6 above, $arg(F_0/x_0) = 90^\circ$.

For convenience, we define a 45° bandwidth such that at the limiting frequencies,

$$arg(F_0/x_0) = \pm 45^\circ \quad \text{w.r.t. that at resonance.}$$

$$\text{i.e. } \tan^{-1}[arg(F_0/x_0)] = \pm 1$$

so, from equation 2.5,

$$\omega_1 A(\rho\eta\omega_1)^{\frac{1}{2}} + \omega_1 R_0 = -\omega_1^2 m - \omega_1 A(\rho\eta\omega_1)^{\frac{1}{2}} + K \qquad 2.9$$

$$\omega_2 R_0 = \omega_2^2 m - K \qquad 2.10$$

eliminating K,

$$\rho\eta = \frac{m^2}{4A^2\omega_1^3}\left[\omega_2^2 - \omega_1^2 - \frac{R_0}{m}(\omega_1 + \omega_2)\right]^2 \qquad 2.11$$

Note: 1. $\frac{m}{A}$ is a constant.

2. Equation 2.10 is independent of $\rho\eta$ ie, ω_2 is constant for a given transducer.

For $\rho\eta = 0$ equation 2.11 becomes:

$$\frac{R_0}{m} = \omega_{20} - \omega_{10} \qquad 2.11(a)$$

substituting for $\frac{R_0}{m}$ in 2.11, and noting that $\omega = 2\pi f$

$$\rho\eta = \frac{\pi m^2}{2A^2 f_1^3}\left[(f_2 + f_1)(f_2 - f_{20} + f_{10} - f_1)\right]^2 \qquad 2.12$$

so $\rho\eta$ can be found by measuring the $\pm 45^\circ$ frequencies. In practice, it proved easier to measure the -3dB bandwidth. It can be shown that for $(\omega_2 - \omega_1)/\omega_2 \ll 1$, as occurs in practice, the -3dB frequencies coincide with the $\pm 45^\circ$ frequencies.

2.6 Finding $\rho\eta$ By Decay Of Free Vibrations.

For this case

$$m\ddot{x} + Z\dot{x} + Kx = 0$$

Solving using D-operators,

$$D = \frac{-Z}{2m} \pm j\left[\frac{K}{m} - \frac{Z^2}{4m^2}\right]$$

Now,

$$Z = R + jX \;,$$

$$(Z/4m)^2 << (K/m)$$

so, using binomial theorem, and considering only positive frequencies,

$$D = \alpha + j\omega$$

where

$$\alpha = \frac{-R}{2m} + \omega_{oo}^3 (RX/4) \qquad 2.13$$

$$\omega \doteqdot \omega_{oo} - (X/2m) \qquad 2.14$$

$$\omega_{oo} = (K/m)^{1/2} \qquad 2.15$$

so, from 2.13,

$$R = \frac{\alpha}{(\omega_{oo}^3 X)/4 - (1/2m)} \qquad 2.16$$

$\rho\eta$ can now be found by combining equations 2.4 and 2.16.

2.7 Comparison Of Measurement Methods.

	DECAY	AMPLITUDE	BANDWIDTH
Instrumentation:			
1. Lab. measurements.	Fair	Simple	Simple
2. Production.	Fair	Simple	Involved
Suitable for light damping:	Good	Good	Fair
Suitable for heavy damping:	No	Good	Fair
Interfacing with transducer:			
1.Single port.	Simple	Critical	Critical
2. Two port.	Simple	Simple	Simple
Constant shear rate possible:	No	Yes	Yes

CHAPTER 3 CHOICE OF TRANSDUCER.

3.1 The Ideal Transducer.

The ideal transducer for viscosity measurement would have the following characteristics:

1. No loss. ie any damping of the transducer would be attributable to the test liquid.

2. Easy to interface with the necessary electronics.

3. Stable transfer function.

4. No longitudinal waves radiated by the transducer into the test liquid.

In addition, it was felt that the transducer should be easy to construct, out of readily available materials.

3.2 Longitudinally Vibrating Transducers.

Transducers of this type were described by Roth and Rich (3) and Taraba (4). Both designs used a magnetostrictive bar as the active element. A coil wound around the bar served as both a driver and as a receiver.

The advantages of such a transducer are:

1. Simple to construct.
2. Fully immersible.
3. Robust.

The disadvantages are:

1. Input and output signals share the same wires.
2. For slurry measurements, the frequency of operation is probably too high.
3. Especially for low viscosity liquids, the longitudinal radiation from this type of transducer causes significant errors.

3.3 Torsional Transducers.

Transducers of this type were described by Mason (1), McSkimin (2), etc.

Most of the designs featured a tubular crystal used both as a driver and as a receiver.

The advantages of such a transducer are:

1. Fully immersible.
2. Generates no longitudinal waves when well designed. (see Ref. 1).
3. Multifrequency operation possible.

The disadvantages are:

1. More difficult to construct than longitudinal transducers.
2. Input and output signals share the same wires.

3. Crystals tend to be fragile.

4. Frequency of operation is probably too high for slurries.

3.4 Vibrating Plate Transducers.

A transducer of this type was described by Woodward (7).

The advantages are:

1. Radiates no longitudinal waves.

2. Has separate driver and pickup, simplifying instrumentation.

3. Simple to construct.

4. Low frequency operation (below 1KHz).

The disadvantages are:

1. Immersion depth critical - disc must be immersed to correct depth.

2. Transfer function not stable - airgap easily disturbed by knocks etc.

3. Will only operate satisfactorily at two or three frequencies, because the mass of the disc inhibits higher resonant modes.

3.5 Flexural Transducers.

A flexural transducer was described by Kremlevskii and Stepichev (8). Apart from its ability to measure the viscosity-density product of a liquid, as with the other transducers described, it can also measure density, allowing viscosity to be calculated.

Further advantages are:

1. Simple to construct.
2. Robust.
3. Generates no longitudinal waves in the liquid.
4. Low frequency operation.

5. Multifrequency operation possible.

6. Depth of immersion less critical than for vibrating plate, because of length of rod.

<u>The disadvantages are</u>:

1. Transfer function not stable - airgap can be disturbed by knocks, etc.

2. Difficult to make fully immersible.

3.6 Transducer Chosen.

As a start, it was decided to try a transducer similar to that of Roth and Rich (3), as the materials were available, except that instead of a flat blade, a thin walled tube was used for greater strength. Initial testing revealed that longitudinal radiation was swamping the shear loading effect in water, so the transducer was modified as suggested by Taraba (4) This eliminated the frontal radiation, but the ring-type vibration induced by Poisson coupling still caused excessive error.

A transducer similar to that of Woodward (7) was then built, and initial tests were promising. It was thus logical to use this transducer in order to develop the neccessary techniques.

This transducer is discussed in detail in section 4.2.

In order to facilitate measurements on slurries, a flexurally vibrating rod similar to that of Kremlevskii and Stepichev (8) was substituted for the vibrating plate described above. This transducer is discussed in detail in section 4.4.

CHAPTER 4. EXPERIMENTAL WORK.

4.1 Introduction.

The experimental work progressed in the following stages:

(1) An Ultrasonic transducer similar to that of Roth and Rich (3) was attempted, but even after effecting the modification used by Taraba (4), it proved unsatisfactory, and was rejected.

(2) A vibrating plate transducer similar to Woodward's (7) was built, and used to develop the necessary measuring technique. It gave good results on homogeneous liquids, but proved unsatisfactory on slurries for the following reasons:

(a) The depth of immersion was critical, so that no stirring of the slurry was possible whilst taking measurements.

(b) A transducer of this type did not operate satisfactorily at more than two frequencies.

This transducer is discussed in detail in section 4.2.1.

(3) A flexurally vibrating rod similar to that of Kremlevskii and Stepichev (8) was built. It operated satisfactorily at four overtones, spanning a frequency range of 10 : 1. Calibration curves of viscous loading at the nth mode were very good (see fig.5.3). Similar curves for slurries were also obtained.

4.2 Vibrating Plate Transducers.

A number of vibrating plate transducers were built and tested on homogeneous Newtonian liquids spanning a viscosity range of nearly three decades.

Early readings were made using amplitude measurement (see 2.4), but inconsistent results were obtained because the airgap of the magnetic drive was easily disturbed.

To circumvent this problem, all subsequent readings were made using bandwidth measurement (see 2.5), which gave repeatable results.

The test liquids used were a number of lubricating oils (see app.A)and water-glycerine solutions. The latter were hygroscopic, so in order to get consistent readings, their viscosities were always measured on a Redwood viscometer immediately before testing.

All the measurements were done at room temperature, so that steady temperature conditions were assumed. A thermometer monitored the liquid temperature to ensure that it remained constant for the duration of the test.

A number of readings were taken for each liquid, the transducer being removed from the liquid and then repositioned between readings. This was done to remove any bias in the readings resulting from incorrect immersion of the transducer.

4.2.1 Description of a Vibrating Plate Transducer.

The transducer (see fig.4.1)comprised a flat disc vibrating in its own plane on the end of a steel reed, the latter anchored to a PZT crystal which served as a pickup.

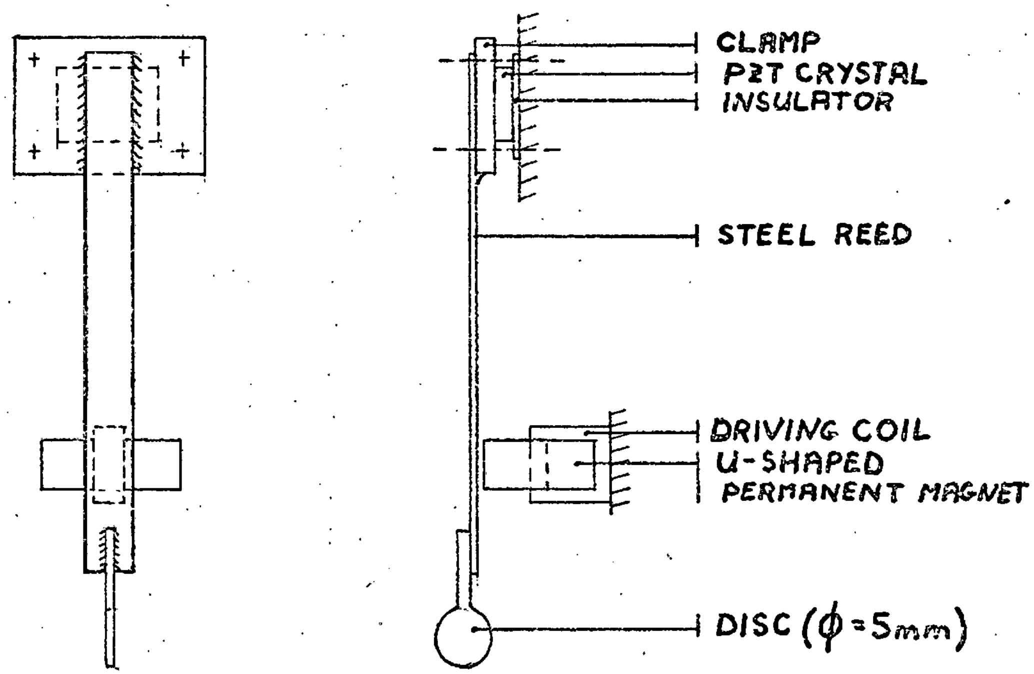
A coil of fine wire wound on a permanent magnet was used to drive the reed from a signal generator. A typical transducer had a Q-factor of approximately 300+ in air, falling to approximately 10 with the disc immersed in the most viscous test oil used. Usually, the fundamental mode and the first overtone were the only satisfactory modes of operation because the mass of the disc suppressed the higher modes. It was neccessary to ensure that no spurious resonances occurred in the transducer and its mountings.

4.2.2 Instrumentation for Bandwidth Measurement.(fig.4.2)

The transducer was driven using a hand-tuned sinewave generator. The resulting output signal from the transducer was amplified, and high-pass filtered to reduce the hum component. This signal was then fed via a 3dB attenuator, which could be switched in or out, to:

1. A measuring amplifier with a meter.
2. An oscilloscope.
3. A frequency counter.

FIG. 4.1. DIAGRAM OF VIBRATING PLATE TRANSDUCER.



4.2.3 Operation.

With the 3dB attenuator in circuit, the drive frequency to the transducer was increased slowly by hand. As resonance was passed, the output voltage would pass through a maximum, this value being noted on the meter of the measuring amplifier.

The 3dB attenuator was then switched out, and the frequency varied until the same meter reading as before was obtained, this corresponding to a -3dB frequency. The frequency was read off the frequency counter, then the same procedure was used to find the other -3dB frequency.

The loading on the transducer was calculated using equation 2.12, and plotted against viscosity-density product for the liquid on a log-log scale. (fig.5.1).

4.3 Measurement of Slurries.

All the measurements made on slurries were intended to show up any dependence of transducer loading on penetration depth for constant viscosity.

The approach first used was to compare the readings obtained using the vibrating plate transducer with those of a standard viscometer. The repeatability of this method was so poor as to render the results unusable. This was due mainly to:

1. The high rate at which the slurry settled out.

2. The lack of a suitable standard viscometer. The orifice of the Redwood clogged easily, making repeatable readings difficult.

A better approach was to utilize the fact that penetration depth is a function of frequency. So, if transducer loading was measured as a function of frequency, any deviation from the expected curve could be attributed to insufficient penetration. However, in order for this to be true, the behaviour of the slurry must be Newtonian. This generally requires that the slurry be dilute. (14)

It was felt that the best way to achieve the measurement would be to use a transducer capable of operating at several frequencies spanning a wide band.

The transducer of 4.2 could not be induced to operate properly at more than two overtones. This was mainly due to the mass of the disc at the end of the reed, which suppressed the higher resonant modes.

4.4 Flexurally Vibrating Rod. (See fig.4 .3).

This transducer was built to overcome the problems discussed in (4.3). Because the rod was uniform, with no lumped mass at the end, it has many strong resonant modes. Bandwidth measurement was initially used with acceptable results, but was found to be too slow for slurry measurements. As a result, amplitude measurement was used.(see 2.4).The airgap was stabilized by fitting a guard ring around the rod to prevent accidental bending at the mount. The transducer was initially calibrated using the approach outlined in (4.2). The second, third and fourth overtones were then calibrated against the first. The advantage of this comparative approach is that the actual viscosity of the test liquid is unimportant, provided it stays constant for the duration of the measurement.

The calibration curves obtained using this method are very good (Fig. 5.3).

4.4.1 Description of Flexurally Vibrating Rod Transducer.

This transducer is very similar to the vibrating plate viscometer described in section 4.2.1. The reed and disc were replaced by a cylindrical rod, part of which is immersed in the test liquid. The Q at all the modes used is approximately 600 in air, and greater than 20 in the most viscous liquid tested. The four usable modes are the second, third, fourth and fifth. The fundamental mode couples strongly to the mounting apparatus, resulting in unreliable measurements. Modes higher than the fifth are either excessively lossy, or near to spurious resonances in the mountings.

In order to allow repeatable readings using amplitude measurement to be made, a guard ring was fitted around the rod to prevent accidental bending at the mount (see fig. 4.3).

4.4.2 Instrumentation (See fig. 4.4)

A sinewave generator is used to excite the immersed transducer sinusoidally at a resonant frequency.

The resulting output voltage from the crystal pickup is amplified by a preamplifier, and high-pass filtered through a filter to reduce vibration pickup at the fundamental frequency of the rod. (39HZ). The filtered signal is connected to:

1. A digital multimeter, to read the A.C. voltage.
2. An oscilloscope, to observe the waveform.
3. A frequency counter.

Operation

The frequency of the function generator is adjusted to give a maximum output signal from the transducer, corresponding to resonance.

Readings in air are taken before and after an experimental run, in order to establish the transducer loss.

The transducer loading is then calculated from equation 2.8(a), and plotted against viscosity-density product for the liquids on a log-log scale. (fig.5.2). Graphs of loading at the various overtones versus loading at the first overtone were plotted (fig.5.3) for use as calibration curves.

4.5 Quartz Slurries. - Measurement with Rod.

It is well-known that these slurries are non-Newtonian. (Reference 14). The extent of the deviation from Newtonian behaviour generally increases with increasing solid concentration. As the immediate aim with slurry measurement was to gauge whether the particulate nature of the slurry gave rise to erroneous readings, the first quartzite-water slurry tested was fairly dilute. The solid volume fraction was 0,065 , corresponding to a density of 1,11 g/cm .

Experimental Procedure.

Because of its low viscosity and the high density of the quartzite this slurry tended to settle out very rapidly, making repeatable measurements difficult. In order to minimize the error, remembering that only comparative readings were of interest, the following method was adopted:

1. The slurry was stirred vigorously using a motor-driven impeller immersed in the slurry for the duration of the test.

2. After a minute or so, the motor was switched off. A fixed time later, (usually fifteen seconds) a reading was taken. In this way, it was hoped that the slurry would always be in a similar state when the reading was taken.

Several readings were taken for each measurement, using the apparatus of section 4.4. Measurements were made at several shear rates at each of the four resonant modes of the transducer. The variation of shear rate was achieved by varying the drive signal to the transducer. It can be shown (app. C) that for a Newtonian liquid, the shear rate is proportional to the output voltage of the transducer, independent of frequency.

The repeatability of the readings thus obtained was only about +10%, but this was considered adequate for the purpose, as the graph plotted (fig. 5.4) demonstrated clear trends.

Similar sets of readings were then taken for quartzite-water slurries with volume fractions of 0,24 and 0,34 , corresponding to densities of 1,41g/cc and 1,58g/cc respectively.

Using the calibration curves (fig. 5.3) found in section 4.4, the transducer loading at all the overtones used was converted to an equivalent loading at the first overtone, as this facilitated comparison of the results.

CHAPTER 5 Discussion Of Results.

5.1 Results Obtained for Vibrating Plate Transducer.

Results obtained using the transducer discussed in 4.2 are plotted in fig. 5.1.

This curve agrees closely with that predicted by the theory derived in section 2.5.

Note:

1. That the viscous loading of the transducer varies as the square root of the viscosity, as expected.

2. The upper -3dB frequency is approximately constant, as predicted by equation 2.10.(see app. D)

3. The use of a simple mathematical model to describe the transducer is justified on the basis of the experimental results.

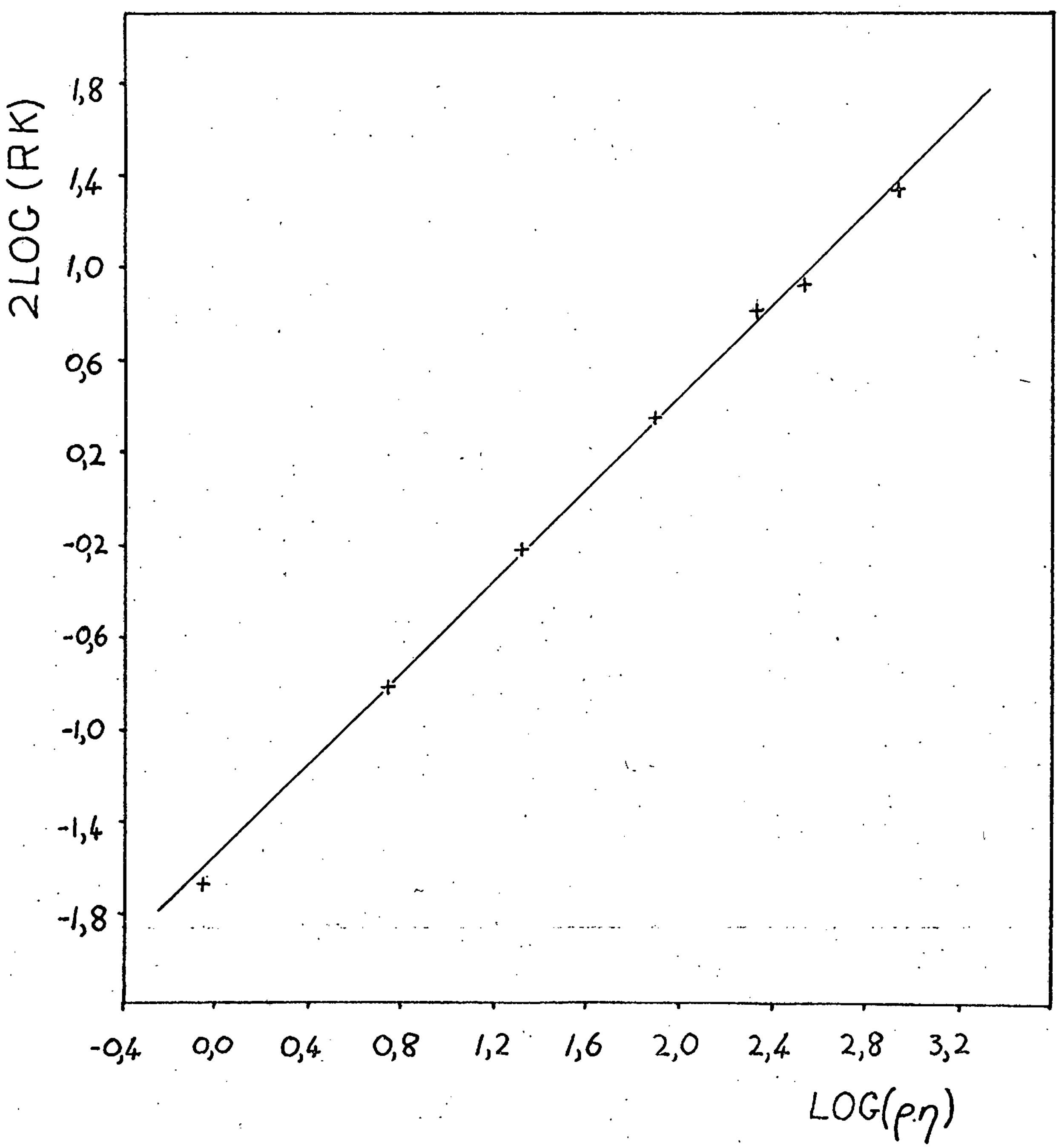
5.2 Results Obtained for Flexurally Vibrating Rod. (see 4.4)

The transducer loading at all the overtones used was plotted against the viscosity-density product of the test liquids. (fig. 5.2). Although the results followed the expected trend, they were disappointing, the points being somewhat scattered.

However, when the loading at the second, third and fourth overtones was plotted against that at the first overtone, the correlation was excellent. (fig.5.3) From this it was deduced that the poor results of fig. (5.2) were due to errors in the determination of the liquid viscosities (ie poor temperature control, in this case.)

It will be noticed that the slope of the graph for the second overtone in fig.(5.3) is steeper than the others. This is due to a spurious resonance in the transducer assembly, which caused the graph to deviate from the expected curve.

FIG. 5.1. PLOT OF LOADING vs $\rho\eta$ FOR 50Hz VIBRATING PLATE AT FIRST OVERTONE.



This overtone was still calibrated and used because, despite the unexpected graph, the results were very repeatable, and the system was useful for evaluating slurries.

5.3 Results on Quartz Slurries.

The loading on the transducer was calculated from equation 2.4.

Using the calibration curves of fig. (5.3), all the readings were then converted to a common base, this being the equivalent loading that would be experienced at the first overtone for a Newtonian liquid. The resultant values were then plotted against log(output voltage). It can be shown (see app. C) that, for a Newtonian liquid, this output voltage is proportional to the shear rate, independent of frequency.

Referring to fig (5.4), it can be seen that, for the dilute slurry, the measurement is independent of both shear rate and frequency within experimental error. (i.e. its behaviour is Newtonian.) This graph also suggests that, at all the frequencies used, the discrete nature of the slurry does not disturb the readings.

Referring to figs. 5.5, 5.6 , it is apparent that these slurries are markedly non-Newtonian, exhibiting shear-thinning bahaviour at low frequencies. This is in agreement with results obtained by Levy (14) on similar slurries.

More interesting is the change in the curves with frequency, for the more dilute of the two slurries, only the fourth overtone exhibits behaviour markedly different from that of the first overtone. For the denser slurry, the third overtone also deviates from the first, while the fourth deviates still further. Note that for very low shear rates, all the curves converge.

It is apparent that this deviation with frequency increases with increasingly non-Newtonian behaviour. (i.e with increasing density).

This is supported by the following observations:

1. At low shear rates, all the curves converge. This might be expected, because the slurry becomes increasingly Newtonian at low shear rates.

2. The rate of deviation with frequency is greatest for the slurry of highest density.

3. There is no observable deviation for the dilute (Newtonian) slurry.

CONCLUSIONS.

1. The theory for viscous loading of a vibrating transducer is valid for the transducers used.

2. At the frequencies of vibration used, for the slurry tests, the particle nature of the slurries does not give rise to erroneous readings.

3. For the slurries tested, the apparent viscosities are a function of frequency. This function depends on the degree of deviation from Newtonian behaviour. It is apparent that if a measurement of steady flow viscosity is desired, a sufficiently low frequency measurement is necessary.

4. The transducer is therefore capable of measuring apparent viscosities of slurries. This will enable a detailed investigation to be carried out of slurry viscosity as a function of frequency.

5. The transducer described could be made part of a self-oscillating loop, so that operation at resonance is assured. This will simplify the measurement.

REFERENCES:

1. MASON W.P.

   Measurement of the Viscosity and Shear Elasticity of Liquids by means of a Torsionally Vibrating Crystal
   A.S.M.E. Transactions, May, 1947.

2. Mc SKIMIN H.J.

   Measurement of Dynamic Shear Viscosity and Stiffness of Viscous Liquids by means of Travelling Torsional Waves

   Journal of the Acoustical Society of America
   Vol. 24, number 4. July 1952.

3. ROTH W. and RICH S.R.

   A New Method for Continuous Viscosity Measurement. General Theory of the Ultra-Viscoson.

   Journal of Applied Physics.
   Vol. 24, number 7. July 1953.

4. TARABA, O.

   Some thoughts on viscosity measurement with ultrasonic transducers

   Ultrasonics, October,1967, page 246.

5. BRADFIELD, G.

   The Application of Piezo-Magnetic Alloys to Ultrasonic Viscometers Operating at High Temperatures and Pressures

   Institution of Electrical Engineers
   Monograph No. 521M. May 1962.

6. RUTTLE S.G.R. and STEPHENSON M.I.

   A High-Pressure Ultrasonic Viscometer
   Ultrasonics International 1975, Conference Proceedings.

7. WOODWARD, J.G.

   A Vibrating-Plate Viscometer

   Journal of the Acoustical Society of America
   Vol. 25, No. 1. January 1953.

8. KREMLEVSKII V.P. and STEPICHEV A.A.

Measurement of the Viscosity and Density of a Liquid by means of a Vibrating Transducer

Soviet Physics - Acoustics
Vol. 21, No. 7. July-August 1975.

9. KOGAN I.N.

Tubular Vibrator with Lumped Masses at the Ends as a Viscometer.

Soviet Physics - Acoustics
Vol. 14, No. 4. April - June 1969.

10. KRUTIN V.N. and SMIRNITSKII I.B.

Measurement of the Viscosity of Newtonian Fluids by means of Vibratory Probes.

Soviet Physics - Acoustics
Vol. 12, No. 1. July - September 1966.

11. SEARS, F.W. and ZEMANSKY, M.W.

College Physics. Third Edition. Addison Wesley Publishing Company (1964).

12. ADAMS, L.F.

Engineering Measurements and Instrumentation.

English Universities Press Ltd. (1975).

13. HINDE, A.L. and LLOYD, P.J.D.

State of the Art of Real-Time Particle Size Analysis as applied to the Mill-Hydrocyclone Process.

Chamber of Mines of South Africa. Research Report No. 37/74.

14. LEVY, C.D.

A Rheological Investigation of Three South African Gold Mine Pulps.

M.Sc. Thesis, Department of Mechanical Engineering, University of Cape Town, 1974.

15. Mc LACHLAN, N.W.

Theory of Vibrations. Dover Publications (1951).

APPENDIX A. Newtonian Oils Used For Calibration.

OIL (Shell)	DENSITY (g/cm^3) (20^oC)	KINEMATIC VISCOSITY (cS) 37,8 °C	98,9 °C
Carnea 21	0,866	22,3	4,25
Turbo 29	0,870	48,0	6,62
Tellus 41	0,887	105	11,41
Tellus 69	0,891	158	14,5
Macoma R75	0,919	335	21,8

Note:

1. The temperature coefficient of the density ρ is very small. Consequently, the above values were used throughout.

2. To convert from kinematic viscosity μ ($m^2 s^{-1}$) to dynamic viscosity η ($g\ cm^{-1} s^{-1}$),

$$\eta = \rho\mu \times 10^4$$

3. The above values are plotted on a "REFUTAS" chart, which has a scale so arranged that the viscosity-temperature graphs are all straight lines, so that they are completely specified by the two values given.

APPENDIX B.

Sonic Theory Of Vibrating Viscometers. (Ref.1).

When a thin large plate immersed in a medium oscillates in its own (x-z) plane, along the x-axis, a shear wave is propagated into the medium along a direction normal to the plate surface (y-axis).

Neglecting edge effects (because the plate is large), the equation of motion of the shear wave is

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{\mu}{\rho_0}\frac{\partial^2 \xi_x}{\partial y^2} \qquad \text{B.1}$$

where ξ_x is the particle displacement in the x-direction.

μ is the shear rigidity of the medium.

ρ_0 is the density of the medium.

Considering only sinusoidal oscillations, if the medium is a Newtonian fluid, we have

$$\mu = j\omega\eta \qquad \text{B.2}$$

where

η is the coefficient of viscosity for the fluid.

ω is the angular frequency of oscillation.

Also, for sinusoidal oscillation, the particle velocity.

$$u_x = j\omega\xi_x \qquad \text{B.3}$$

Combining equations B.1, B.2 and B.3 gives

$$\frac{\partial u_x}{\partial t} = \frac{\eta}{\rho_0}\frac{\partial^2 u_x}{\partial y^2} \qquad \text{B.4}$$

Solving this gives

$$u_x = u_0 e^{j\omega t} \exp[-(\pi f \rho_0/\eta)^{1/2}(1+j)y] \qquad \text{B.5}$$

where u_0 is the amplitude of oscillation of the plate.

It can be seen that the wave is attenuated very rapidly with y , by a factor

$$\exp[-(\pi f \rho_0/\eta)^{\frac{1}{2}} y]$$

It will now be shown that the wave is attenuated to approximately 1 per cent of its initial value within one wavelength of the plate.

Comparing equation B.1 to the general wave equation, propagation velocity

$$c = (\mu/\rho_0)^{\frac{1}{2}}$$

or, substituting equation B.2, for a Newtonian liquid, noting that $j^{\frac{1}{2}} = \frac{1}{\sqrt{2}}(1+j)$,

$$c = (\omega\eta/2\rho_0)^{\frac{1}{2}}(1+j) .$$

so

$$|c| = (\omega\eta/\rho_0)^{\frac{1}{2}} \qquad \text{B.6}$$

Now wavelength $\lambda = 2\pi c/\omega$

Substituting in B.6, noting that $\omega = 2\pi f$,

$$\lambda = (2\pi\eta/f\rho_0)^{\frac{1}{2}} \qquad \text{B.7}$$

Now, from B.5, attenuation coefficient

$$\alpha = (\pi f \rho_0/\eta)^{\frac{1}{2}} \qquad \text{B.8}$$

Equation B.7 divided by B.8 yields

$$\alpha = \sqrt{2}\,\pi/\lambda \qquad \text{B.9}$$

From equation B.5

$$\left|\frac{u_x}{u_0}\right|_{y=\lambda} = e^{-\sqrt{2}\pi} \doteqdot 0{,}012$$

So the shear wave is attenuated to 1,2% of its initial value within a wavelength of the transducer.

For example: In water ($\eta \doteq 10^{-2}$ poise), at 1KHz,

$$\lambda \doteq 80 \mu m$$

The loading of the medium on the plate is found as follows:

By definition of the coefficient of viscosity η,

$$\sigma_S = -\eta \frac{du_x}{dy} \qquad \text{B.10}$$

where σ_S is the shear stress.

Combining equations B.5 and B.8 yields

$$\frac{\sigma_S}{U_x} = (\pi f \rho_0 \eta)^{\frac{1}{2}} (1+j) \qquad \text{B.11}$$

This ratio is the specific shear impedance of the medium, Z_S .

Note:

1. Z_S is complex, the resistive and reactive components being equal for Newtonian liquids.

2. Z_S is a function of frequency.

3. It is not possible to solve for η unless ρ_0 is known.

APPENDIX C.

Theory Of Flexural Transducer. (Refs. 8,15)

In general, the differential equation describing clamped flexural vibration in a bar is:

$$EI\frac{\partial^4 \xi}{\partial x^4} + Z\frac{\partial \xi}{\partial t} + m\frac{\partial^2 \xi}{\partial t^2} = f(x) \qquad \text{C.1}$$

Where

E is Young's modulus for the bar.

I is the moment of inertia of the bar about its neutral axis.

Z is the loading impedance.

m is the mass per unit length of the bar.

$f(x)$ is the forcing function.

$\xi = \xi(x,t)$ is the displacement of the rod at any point.

For $f(x)$ sinusoidal,

$$f(x) = F(x)e^{j\omega t}, \qquad \text{C.2}$$

For simplicity, consider the case when $f(x)$ acts at a point, chosen to be the free end of the bar.

As the system is linear, one may assume a solution

$$\xi(x,t) = \xi(x)e^{j\omega t}$$

Substituting this into C.1 yields

$$EI\frac{d^4 \xi}{dx^4} + \xi(j\omega Z - \omega^2 m) = F(\ell)$$

which can be rewritten

$$\frac{d^4 \xi}{dx^4} - k^4 \xi = \frac{F(\ell)}{EI} \qquad \text{C.3}$$

This is the standard form for flexural vibrations, showing that the resistive loss can be expressed as a complex wave number k,

where

$$k^4 = (\omega^2 m - j\omega Z)/EI \qquad \text{C.3(a)}$$

The solution to this equation is

$$\xi(x,t) = \frac{Fe^{j\omega t}}{2EIk^3}[1+\cosh(k\ell)\cos(k\ell)]^{-1}.$$

$$[(\sinh(k\ell)+\sin(k\ell))(\cosh(kx)-\cos(kx)) \\ -(\cosh(k\ell)+\cos(k\ell))(\sinh(kx)-\sin(kx))] \quad \text{C.4}$$

Of specific interest is the transverse force experienced at $x=0$ (ie. at the mount).

Now, for $EIZ/4\omega m \ll 1$, as occurs in practice, equation C.3(a) can be rewritten (using binomial theorem),

$$k \doteqdot (\omega^2 m/EI)^{1/4}(1-j(EIZ/4\omega m))$$

so, for $Z=R+jX$, a general impedance,

$$k = k_0[1+\frac{EIX}{4\omega m} - j\frac{EIR}{4\omega m}] \quad \text{C.5}$$

where $k_0=(\omega^2 m/EI)^{1/4}$ is the wave number with $Z=0$, ie in the absence of loading.

Now shearing force at any point is

$$F(x) = EI\frac{\partial^3\xi}{\partial x^3}$$

so, differentiating equation C.4 three times, and setting $x=0$,

$$F(0) = Fe^{j\omega t}\frac{(\cosh(k\ell)+\cos(k\ell))}{1+\cosh(k\ell)\cos(k\ell)} \quad \text{C.6}$$

It is apparent that resonance occurs for

$$1+\mathrm{Real}(\cosh(k\ell)\cos(k\ell))=0 \quad \text{C.7}$$

Substituting C.5 in C.7 yields

$$\cosh(k_n\ell)\cos(k_n\ell) \doteqdot -1$$

where $k_n=k_0(1+\frac{EIX}{4\omega_n m})$ is the wave number at the nth resonant mode.

Substituting C.5 and C.7 in C.6, and noting that

$$\cos(k_n\ell) \ll 1$$

$$(EIR/4\omega m) \ll 1$$

we get that, at resonance,

$$\frac{F(O)}{Fe^{j\omega t}} \doteqdot j\frac{4\omega m K_n}{k_n \ell EIR} \qquad \text{C.8}$$

$$K_n \doteqdot \left[\frac{1-\frac{3}{2}\cos^2(k_n\ell)}{1-\cos(k_n\ell)}\right]$$

ie. the relative magnification at resonance is

$$\frac{F(O)}{F} = \frac{K_n \omega_o}{k_n \ell R} \qquad \text{C.9}$$

For a mass-spring system, the relative magnification is

$$R.M. = \frac{\omega_o m}{R} \qquad \text{C.9(a)}$$

Comparing C.9 and C.9 (a), it can be seen that the rod behaves as a mass-spring system at resonance.

Kremlevskii & Stepichev (8) have shown that the load presented to a cylinder vibrating transversely in a Newtonian liquid is:

$$Z = \pi r\ell[2(2\omega\rho\eta)^{1/2} + j(\omega\rho r + 2(2\omega\rho\eta)^{1/2}] \qquad \text{C.10}$$

where

r is the radius of the rod.
ℓ is the length of the rod.
ρ is the density of the liquid.
η is the viscosity of the liquid.

In practice, the transducer is not lossless, so that in the absence of a liquid, $Z = R_o$.

Also, the rod is not fully immersed. For light damping, the small load imposed by the liquid can be distributed uniformly over the entire length of the rod, but with regard to the load factor $\beta \leq 1$.

so

$$Z = R_o + \pi r\ell\beta[2(2\omega\rho\eta)^{1/2} + j(\omega\rho r + 2(2\omega\rho\eta)^{1/2}] \qquad \text{C.11}$$

since $Z = R + jX$,

$$R = R_o + 2\pi r \ell \beta (2\omega\rho\eta)^{1/2} \qquad \text{C.12}$$

$$X = \pi r \ell \beta (\omega\rho r + 2(2\omega\rho\eta)^{1/2}) \qquad \text{C.13}$$

Note that if R and X are measured simultaneously, it is possible to solve for ρ and η separately, if the liquid is Newtonian.

The Relationship Between Output Voltage And Amplitude Of Vibration.

The pickup crystal generates a voltage V_o proportional to the compressive force applied to it, which, in this case, is the shear force in the rod at $x = 0$. It is desirable to relate V_o to the amplitude ξ_o of the rod vibration, in order to estimate shear rate.

Substituting for $x = \ell$ in C.4, C.14

$$\xi(\ell,t) = \frac{Fe^{j\omega t}}{2Ak_n^3}\left[\frac{\sin(k_n\ell)\cosh(k_n\ell) - \sinh(k_n\ell)\cos(k_n\ell)}{1 + \cosh(k_n\ell)\cos(k_n\ell)}\right]$$

C.14 divided by C.6 yields

$$\frac{\xi(\ell,t)}{F(0,t)} = \frac{1}{Ak_n^3}\left[\frac{\sin(k_n\ell)\cosh(k_n\ell) - \sinh(k_n\ell)\cos(k_n\ell)}{\cosh(k_n\ell) + \cos(k_n\ell)}\right] \qquad \text{C.15}$$

Referring to table (C.1), $|\cos(k_n\ell)| < 0{,}02$, for $n \geq 2$

so $$\frac{\xi(\ell,t)}{F(0,t)} \doteq \frac{1}{Ak_n^3}$$

Substituting C.5 for k_n, noting that V_o is proportional to $F(0)$,

$$\frac{\xi(x,t)}{V_o(t)} \propto \ell \cdot \omega^{-3/2}$$

Now, for sinusoidal oscillation, velocity

$$u(x,t) = j\omega\xi(x,t)$$

so

$$\frac{u(x,t)}{V_o(t)} \propto \ell \ \omega^{-\frac{1}{2}}$$

Also, from equation B.5, shear rate

$$\frac{\partial u(x,t)}{\partial y} \propto \ell \, (\rho\omega/\eta)^{\frac{1}{2}}$$

so

$$\frac{(\partial u(x,t))/\partial y}{V_o(t)} \propto \ell \, (\rho/\eta)^{\frac{1}{2}} \qquad \text{FOR } n \geqslant 2$$

i.e. independent of frequency.

Table C.1

n	cos $K_n l$
1	-0,300
2	-0,0184
3	~0
4	~0
5	~0

APPENDIX D

TABLE OF READINGS TAKEN USING A VIBRATING PLATE TRANSDUCER. (Plotted in fig. 5.1.)

Key to headings.

ρ = Density of liquid. (g cm^{-3}).

η = Viscosity of liquid. (cp).

KR = Transducer loading calculated using equation 2.12, where

K is a constant of proportionality.

R = A $(\rho\eta)^{1/2}$ (see equation 2.4).

f_1 = lower -3dB frequency. (Hz)

f_2 = upper -3dB frequency. (Hz)

T = Temperature of liquid. (^oC)

LIQUID	T	f_1	f_2	LOG$(KR)^2$	LOG $\rho\eta$
None		355,3	357,6		
Tapwater	25	353,3	357,0	-1,65	-0,051
Water-Glycerine:					
3:2	21	351,1	357,1	-0,80	0,748
1:2	22	347,1	356,6	-0,21	1,32
1:4	23	340,1	356,0	0,36	1,90
Oils:					
Tellus41	22	330,0	354,7	0,82	2,33
Tellus69	22	325,9	353,3	0,92	2,52
Macoma R75	22	308,1	348,7	1,34	2,93

Note:

1. Viscosities of the oils used were

obtained as per appendix A.

2. Viscosities of the water-glycerine solutions were measured on a Redwood orifice viscometer.

APPENDIX E

TABLES OF READINGS TAKEN USING FLEXURALLY VIBRATING ROD TRANSDUCER.

E.1 Readings For Newtonian Oils Listed In Appendix A. (Plotted in figs. 5.2 and 5.3).

Key to headings.

$\boxed{T}$	Liquid temperature (^{o}C).
$\boxed{f_{oo}}$	Transducer resonant frequency in air (Hz)
$\boxed{f_{o}}$	Transducer resonant frequency in liquid (Hz)
$\boxed{V_{io}}$	Transducer input voltage in air (dB re OdB)
$\boxed{V_{oo}}$	Preamplifier output voltage in air (volts)
$\boxed{V_{i}}$	Transducer input voltage in liquid (dB re OdB)
$\boxed{V_{o}}$	Preamplifier output voltage in liquid (volts)
$\boxed{\text{GAIN}}$	Preamplifier gain (dB)
$\boxed{\text{LOG } K_n R}$	Transducer loading at the nth overtone calculated using equation 2.8 (a),

where

K_n is the constant relating to the nth overtone.

$$R = A(\rho\eta)^{1/2} \qquad \text{(see equation 2.4)}$$

$\boxed{\text{LOG } \rho\eta}$ Liquid viscosity - density product. (see Appendix A.)

CARNEA 21

T	f_{oo}	f_o	V_{io}	V_{oo}	V_i	V_o	GAIN	LOG K_nR	LOG $\rho\eta$
20	279,1	273	-30	2,35	-10	2,53	40	-0,953	1,61
	785,2	769	-20	1,83	0	1,35	40	-0,164	
	1533,9	1502	-20	3,06	0	4,17	40	-0,684	
	2520,9	2480	-10	2,56	0	2,10	40	-0,453	
20		273			-20	0,79	40	-0,946	
		769			-10	0,418	40	-0,154	
		1502			-20	0,414	40	-0,680	
		2480			-20	0,206	40	-0,441	
16		273			-20	0,75	40	-0,921	
		769			0	1,25	40	-0,128	
		1503			0	3,90	40	-0,650	
		2482			0	1,98	40	-0,418	
17		273			-10	2,39	40	-0,925	1,68
		769			-10	0,397	40	-0,130	
		1503			-10	1,24	40	-0,653	
		2482			-10	0,63	40	-0,422	
18		273			-10	2,49	40	-0,945	1,65
		769			0	1,31	40	-0,150	
		1503			-10	1,28	40	-0,669	
		2482			-10	0,65	40	-0,440	

TURBO 29

T	f_{oo}	f_o	V_{io}	V_{oo}	V_i	V_o	GAIN	LOG $K_n R$	LOG $\rho\eta$
18	278,9	273	-30	2,09	-10	1,71	40	-0,770	2,06
	785	768	-20	1,58	0	0,89	40	0,025	
	1533	1501	-20	2,71	0	2,85	40	-0,503	
	2519	2480	-10	2,34	0	1,51	40	-0,278	
18		273			-10	1,70	40	-0,768	
		767			-10	0,277	40	0,032	
		1501			-20	0,284	40	-0,501	
		2480			-20	0,150	40	-0,274	
18		273			-30	0,171	40	-0,770	
		768			0	0,90	40	0,020	
		1502			0	2,88	40	-0,508	
		2480			0	1,53	40	-0,285	
19		273			-10	1,74	40	-0,779	2,03
		768			0	0,91	40	0,015	
		1501			0	2,91	40	-0,513	
		2480			0	1,54	40	-0,289	
19		273			-10	1,75	40	-0,781	
		768			0	0,90	40	0,020	
		1501			0	2,94	40	-0,518	
		2480			0	1,54	40	-0,289	

TELLUS 41

T	f_{oo}	f_o	V_{io}	V_{oo}	V_i	V_o	GAIN	LOG $K_n R$	LOG $\rho\eta$
19	279,0	271	-30	2,17	0	3,85	40	-0,611	2,41
	785,2	766	-20	1,81	0	0,58	40	0,222	
	1534,0	1498	-20	3,06	0	1,97	40	-0,323	
	2520,9	2477	-10	2,57	0	1,08	40	-0,095	
19		272			-100	1,23	40	-0,616	
		765			-10	0,187	40	0,213	
		1499			-10	0,63	40	-0,329	
		2477			-10	0,342	40	-0,096	
		272			-20	0,379	40	-0,603	
		765			0	0,58	40	0,222	
		1498			0	1,98	40	-0,326	
		2477			0	1,08	40	-0,095	
19		272			0	3,89	40	-0,615	
		766			-10	0,184	40	0,221	
		1499			-10	0,63	40	-0,329	
		2478			-10	0,341	40	-0,095	
17		271			0	3,46	40	-0,562	2,49
		766			0	0,54	40	0,254	
		1498			0	1,83	40	-0,289	
		2478			0	1,01	40	-0,062	

TELLUS 69

T	f_{oo}	f_o	V_{io}	V_o	V_i	V_o	GAIN	LOG $K_n R$	LOG $\rho\eta$
18	279,0	270	-30	1,97	0	2,85	40	-0,475	2,67
	785,2	763	-20	1,67	0	0,420	40	0,366	
	1534,0	1496	-20	2,86	0	1,50	40	-0,199	
	2521,1	2474	-10	2,42	0	0,83	40	0,031	
18		270			0	2,86	40	-0,477	
		763			0	0,426	40	0,359	
		1496			0	1,51	40	-0,203	
		2475			0	0,83	40	0,031	
18		271			-20	0,284	40	-0,474	
		763			-10	0,137	40	0,352	
		1497			-20	0,150	40	-0,199	
		2474			-10	0,264	40	0,028	
18		271			-10	0,91	40	-0,480	
		763			0	0,430	40	0,355	
		1496			-10	0,479	40	-0,204	
		2474			0	0,84	40	0,025	
19		271			0	2,93	40	-0,488	2,64
		763			0	0,439	40	0,346	
		1496			0	1,54	40	-0,212	
		2474			0	0,85	40	0,019	

MACOMA R75

T	f_{oo}	f_o	V_{io}	V_{oo}	$\dot{V}_i$	V_o	GAIN	LOG $K_n R$	LOG ρ7
27	278,7	269	-30	2,04	0	2,51	40	-0,417	2,78
	785	760	-20	1,54	0	0,36	40	0,433	
	1533	1491	-20	2,67	0	1,31	40	-0,139	
	2518	2467	-10	2,33	0	0,72	40	0,098	
27		269			0	2,47	40	-0,410	
		759			0	0,36	40	0,433	
		1490			0	1,29	40	-0,132	
		2466			0	0,71	40	0,105	
20		268			0	1,91	40	-0,294	3,02
		756			0	0,264	40	0,571	
		1487			0	0,98	40	-0,007	
		2463			0	0,55	40	0,226	
17	278,9	267	-30	2,05	0	1,53	40	-0,195	3,15
	785	753	-20	1,66	0	0,217	40	0,658	
	1534	1485	-20	2,82	0	0,83	40	0,068	
	2520	2461	-10	2,38	0	0,459	40	0,311	
17		267			0	1,53	40	-0,195	
		753			0	0,215	40	0,662	
		1484			0	0,85	40	0,057	
		2462			0	0,460	40	0,310	

E.2 Readings For Quartz Slurries.

Key to headings (not given in E.1)

$\boxed{\rho}$ Density of slurry (g cm^{-3})

$\boxed{V_o\text{-dB}}$ Preamplifier output voltage (dB re 1 volt at 40 dB GAIN).

$\boxed{\text{LOG } K_nR \text{ - Relative}}$ Actual Transducer loading at nth overtone converted to equivalent loading that would be experienced at the first overtone for a Newtonian liquid. (obtained using fig. 5.3).

$\boxed{\rho = 1{,}11}$

V_i	GAIN	V_o VOLTS	V_o dB	LOG $K_n R$ ACTUAL	LOG $K_n R$ RELATIVE
-40	60	2,40	-12,4	-1,594	-1,594
		2,40			
		2,40			
-30	40	0,80	-1,94	-1,633	-1,633
		0,80			
		0,79			
-20	40	2,30			
		2,31	7,27	-1,573	-1,573
		2,31			
-10	20	0,68	16,7	-1,508	-1,508
		0,71	17,0	-1,536	-1,536
		0,67			
		0,68			
-15	40	4,18	12,4	-1,567	-1,567
		4,16			
		4,21			
		4,22			

f_{oo}	= 278,9
V_{oo}	= 1,91
V_{io}	= -30
T	= 21
f_o	= 273

$\boxed{\rho = 1{,}11}$

V_i	GAIN	V_o VOLTS	V_o dB	LOG $K_\eta R$ ACTUAL	LOG $K_\eta R$ RELATIVE
-30	60	1,32	-17,6	-0,753	-1,50
		1,31			
		1,32			
-20	60	4,19			
		4,21			
		4,19			
		4,23	-7,47	-0,765	-1,50
		4,17			
-10	40	1,40			
		1,39	2,86	-0,777	-1,52
		1,38			
0	40	4,35			
		4,31	12,7	-0,772	-1,51
		4,31			
		4,32			
-15	40	0,77	-2,27	-0,768	-1,51
		0,76			
		0,77			

$f_{oo} = 785$

$V_{oo} = 1{,}59$

$V_{io} = -20$

$T = 21$

$f_o = 768$

$\boxed{\rho = 1{,}11}$

V_i	GAIN	V_o VOLTS	V_o dB	LOG $K_n R$ ACTUAL	LOG $K_n R$ RELATIVE
-30	60	3,91	-8,16	-1,352	-1,59
		3,91			
		3,92			
-20	40	1,22			
		1,21	1,66	-1,340	-1,58
		1,21			
-10	40	3,92	11,9	-1,337	-1,58
		3,87			
		3,94			
0	20	1,15	21,2	-1,296	-1,54
		1,16			
		1,16			
-15	40	2,25	7,04	-1,352	-1,59
		2,25			
		2,25			

$f_{oo} = 1533$

$V_{oo} = 2,75$

$V_{io} = -20$

$T = 21$

$f_o = 1499$

$\boxed{\rho = 1{,}11}$

V_i	GAIN	V_o VOLTS	V_o dB	LOG $K_\eta R$ ACTUAL	LOG $K_\eta R$ RELATIVE
-30	60	1,35	-17,4	-0,992	-1,49
		1,36			
		1,35			
-20	60	4,14	- 7,66	-0,963	-1,46
		4,14			
		4,15			
		4,12			
-15	40	0,77	-2,27	-0,983	-1,48
		0,77			
		0,76			
-10	40	1,35	2,61	-0,970	-1,47
		1,35			
		1,33			
0	40	4,25	12,6	-0,987	-1,49
		4,26			
		4,26			

$f_{oo} = 2520$

$V_{go} = 2{,}49$

$V_{io} = -10$

$T = 21$

$f_o = 2476$

$\rho = 1,41$

V_i	GAIN	V_o VOLTS	V_o dB	LOG $K_n R$ ACTUAL	LOG $K_n R$ RELATIVE
-10	40	4,87	13,8	-1,31	-1,31
		4,80			
		4,86			
-20	40	1,41			
		1,44			
		1,42	3,05	-1,26	-1,26
-30	60	3,51			
		3,56			
		3,54	-9,02	-1,13	-1,13
-40	60	0,79	-22,0	-0,96	-0,96
		0,83		-0,98	-0,98
		0,78		-0,95	-0,95
		0,78			

$f_{oo} = 278$

$V_{oo} = 1,99$

$V_{io} = -30$

$T = 23$

$f_o = 271$

$\rho = 1,41$

V_i	GAIN	V_o VOLTS	V_o dB	LOG $K_n R$ ACTUAL	LOG $K_n R$ RELATIVE
-20	60	1,53	-16,3	-0,23	-1,02
		1,47		-0,21	
		1,56		-0,24	
		1,46		-0,21	
0	40	2,23	7,0	-0,42	-1,19
		2,24			
		2,22			

$f_{oo} = 783$

$V_{oo} = 1,55$

$V_{io} = 20$

$T = 23$

$f_o = 761$

$\boxed{\rho = 1,41}$

V_i	GAIN	V_o VOLTS	V_o dB	LOG $K_\eta R$ ACTUAL	LOG $K_\eta R$ RELATIVE
-10	40	2,15	6,65	-0,96	-1,21
		2,17			
		2,17			
-20	40	0,62			
		0,61	-4,3	-0,90	-1,16
		0,61			
-30	60	1,66			
		1,60			
		1,62	-15,8	-0,80	-1,07
		1,64			
-40	60	0,44	-27,1	-0,72	-0,98
		0,42		-0,70	
		0,45			

f_{oo}	= 1529
V_{oo}	= 1486
V_{io}	= -20
T	= 23,5
f_o	= 1486

$\boxed{\rho = 1{,}41}$

		V_o		LOG $K_n R$	
V_i	GAIN	VOLTS	dB	ACTUAL	RELATIVE
0	40	1,83	5,25	-0,39	-0,89
		1,83			
		1,80			
-10	40	0,54	-5,35	-0,35	-0,85
		0,53			
		0,55			
-20	60	1,55		-0,29	
		1,59		-0,31	
		1,76		-0,37	
		1,45		-0,26	
		1,62		-0,32	
		1,60	-15,9	-0,31	-0,81
		1,53		-0,29	
-30	60	0,51			
		0,49			
		0,50	- 26	-0,31	-0,81

$f_{oo} = 2514$

$V_{oo} = 2{,}30$

$V_{io} = -10$

$T = 24$

$f_o = 2453$

$\boxed{\rho = 1{,}58}$

V_i	GAIN	V_o VOLTS	V_o dB	LOG K_nR ACTUAL	LOG K_nR RELATIVE
-40	60	0,37	-28,4		
		0,38	-28,4		
		0,38	-28,4	-0,61	-0,61
-30	60	2,37	-12,5	-0,93	-0,93
		2,47			
		2,36			
		2,36			
-20	40	1,16			
		1,11			
		1,15			
		1,13	1,06	-1,14	-1,14
-10	40	4,37	12,8	-1,25	-1,25
		4,22			
		4,18		-1,22	
		4,39			

$f_{oo} = 278$

$V_{oo} = 1{,}99$

$V_{io} = -30$

$T = 23{,}5$

$f_o = 270$

$\boxed{\rho=1,58}$

V_i	GAIN	V_o VOLTS	V_o dB	LOG K_nR ACTUAL	LOG K_nR RELATIVE
-20	60	0,71			
		0,73	-22,7	0,10	-0,40
		0,70			
-10	60	2,20	-13,2	0,11	-0,39
		2,36			
		2,31	-12,7	0,09	-0,41
0	40	0,99	- 0,09	-0,06	-0,57
		1,02			
		0,77	- 2,3	0,07	-0,43
		0,82			
		0,80			

f_{oo}=2514

V_{oo}= 2,3

V_{io}= -10

T = 24

f_o= 2440

$\boxed{\rho=1,58}$

V_i	GAIN	V_o VOLTS	V_o dB	LOG KR ACTUAL	LOG KR RELATIVE
-20	60	0,69			
		0,62	-24,2	0,19	-0,61
		0,70			
		0,73			
-10	60	3,16	-10	-0,03	-0,83
		3,20			
		3,45	- 9,2	-0,07	-0,86
		3,45			
		3,45			
0	40	1,62	4,2	-0,27	-1,05
		1,55	3,8	-0,24	-1,02
		1,57			
		1,57			

f_{oo}= 783

V_{oo}=1,55

V_{io}= -20

T = 24

f_o = 758

$\boxed{\rho = 1,58}$

V_i	GAIN	V_o VOLTS	V_o dB	LOG $K_n R$ ACTUAL	LOG $K_n R$ RELATIVE
-30	60	0,70			
		0,66	-23,6	-0,35	-0,62
		0,70	-23,1	-0,38	-0,65
		0,65			
-20	60	2,52	-12,0	-0,44	-0,71
		2,60			
		2,98	-10,5	-0,53	-0,80
		2,78			
		2,53			
-10	40	1,30			
		1,31	2,3	-0,69	-0,95
		1,29			
0	40	4,60	13,3	-0,74	-1,0
		4,66			
		4,70			
		4,74		-0,76	-1,02

$f_{oo} = 1529$

$V_{oo} = 2,69$

$V_{io} = -20$

$T = 24$

$f_o = 1480$